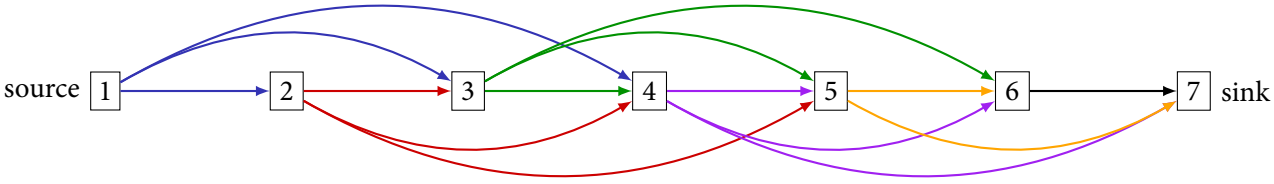
B Solutions to Problems

Solution to Problem 1.



- Node i corresponds to the beginning of year i.
- Edge (i, j) corresponds to purchasing a new machine at the beginning of year i and keeping it until the beginning of year j.
- The length c_{ij} of edge (i, j) is the negative of the revenue associated with purchasing a new machine at the beginning of year i and keeping it until the beginning of year j. In particular,

$$c_{12} = -(4500 - 500 + 3000) = -7000$$
$$c_{13} = -(4500 + 3000 - 500 - 700 + 1800) = -8100$$
$$c_{14} = -(4500 + 3000 + 1500 - 500 - 700 - 1100 + 500) = -7200$$

$$c_{23} = -(-5000 + 4500 - 500 + 3000) = -2000$$
$$c_{24} = -(-5000 + 4500 + 3000 - 500 - 700 + 1800) = -3100$$
$$c_{25} = -(-5000 + 4500 + 3000 + 1500 - 500 - 700 - 1100 + 500) = -2200$$

(Note that starting in year 2, we need to incorporate the cost of buying a new machine.)

$$c_{34} = -(-5000 + 4500 - 500 + 3000) = -2000$$
$$c_{35} = -(-5000 + 4500 + 3000 - 500 - 700 + 1800) = -3100$$
$$c_{36} = -(-5000 + 4500 + 3000 + 1500 - 500 - 700 - 1100 + 500) = -2200$$

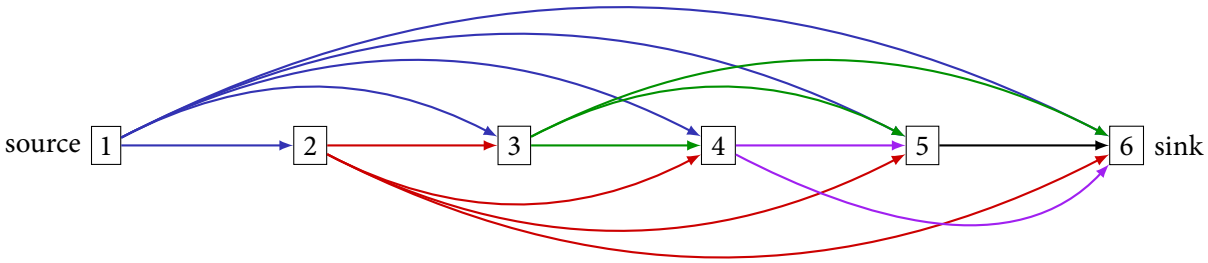
$$c_{45} = -(-5000 + 4500 - 500 + 3000) = -2000$$
$$c_{46} = -(-5000 + 4500 + 3000 - 500 - 700 + 1800) = -3100$$
$$c_{47} = -(-5000 + 4500 + 3000 + 1500 - 500 - 700 - 1100 + 500) = -2200$$

$$c_{56} = -(-5000 + 4500 - 500 + 3000) = -2000$$
$$c_{57} = -(-5000 + 4500 + 3000 - 500 - 700 + 1800) = -3100$$

$$c_{67} = -(-5000 + 4500 - 500 + 3000) = -2000$$

- The negative of the length of a shortest path is the maximum total revenue earned over the 6 year period.
- The nodes in a shortest path tell us when to buy a machine. For example, suppose the edges in a shortest path are $(1, 3), (3, 6), (6, 7)$. Then the company should buy a new machine in years 3 and 6 (note that it already has a new machine at year 1).

Solution to Problem 2.



- Node i corresponds to the beginning of year i.
- Edge (i, j) corresponds to purchasing a new machine at the beginning of year i and keeping it until the beginning of year j.
- The length c_{ij} of edge (i, j) is calculated as follows:

$c_{12} = 170 + 38 = 208$
$c_{13} = 170 + 38 + 50 = 258$
$c_{14} = 170 + 38 + 50 + 97 = 355$
$c_{15} = 170 + 38 + 50 + 97 + 182 = 537$
$c_{16} = 170 + 38 + 50 + 97 + 182 + 304 = 841$

$c_{23} = 190 + 38 = 228$
$c_{24} = 190 + 38 + 50 = 278$
$c_{25} = 190 + 38 + 50 + 97 = 375$
$c_{26} = 190 + 38 + 50 + 97 + 182 = 557$

$c_{34} = 210 + 38 = 248$
$c_{35} = 210 + 38 + 50 = 298$
$c_{36} = 210 + 38 + 50 + 97 = 395$

$c_{45} = 250 + 38 = 288$
$c_{46} = 250 + 38 + 50 = 338$

$c_{56} = 300 + 38 = 338$

- The length of a shortest path is the minimum total cost incurred over the 5 year period.
- The nodes in a shortest path tell us when to buy a machine. For example, suppose the edges in a shortest path are $(1, 2)$, $(2, 5)$, $(5, 6)$. Then the company should buy a new machine in years 1, 2, and 5.